

Vaccination and Disease Eradication: A Dynamic Analysis

Kenea Mideksa, Torben



**Thesis Submitted In Partial Fulfillment of the Requirements for
the Award of Masters of Philosophy in Environmental and
Development Economics,**

**Department of Economics,
University of Oslo,
Oslo, Norway
November, 2005**

ACKNOWLEDGMENT

There are a number of people and organizations whose contributions have been indispensable to this work. It may not be possible to mention them all. But I do not want to pass without mentioning those individuals whose contributions have been central and the work would not have been in the present state without their support.

First and foremost, I would like to pass my gratitude to my supervisors at the department of economics, University of Oslo: Professor Michael Hoel and Atle Seirstad. Especially my interest in the work emerged with the initiative discussion I had with Michael. Beginning from the initiative discussion to the final completion of the work, his role has been crucial. The support and contribution I received from both my supervisors have enriched my research.

I would also like to extend my deepest gratitude to [Health Economic Research Organization] HERO for the financial support provided for this particular research. Thanks HERO for funding and giving this opportunity to such kind of research.

Besides, I am grateful to the Norwegian Quota Program for financing my stay here in Norway to complete my M Phil study in Environment and Development Economics at the University of Oslo.

Moreover, I am thankful to Catharina Vogt and Sara Tadiwos for proofreading the final draft. Finally, I would like to forward my deep gratefulness to Professor Knut Sydseater and Professor Olav Bjerkholt for the crucial support and good environment they created during the course of my research and my stay at the department of Economics.

Table of Contents

ACKNOWLEDGMENT	ii
LIST OF FIGURES	iv
Summary of Vaccination and Disease Eradication: A Dynamic Analysis	1
I. Introduction and Background of the Research	3
II. Is Eradication of Disease Possible?.....	6
III. Dynamic Models of Infectious Diseases and Health Policies	11
<i>Overview of Dynamic Models.....</i>	<i>11</i>
<i>Dynamics in Endemic SIR Model</i>	<i>12</i>
<i>Public Health Policies.....</i>	<i>17</i>
<i>Policy Options</i>	<i>18</i>
IV. Optimal Disease Eradication in Dynamic Framework.....	24
<i>APPLICATION TO POLIO.....</i>	<i>31</i>
VI. Concluding Remarks.....	34

LIST OF FIGURES

Figure 1: Phase Diagrams for Susceptible and Infected Individuals for a constant vaccination

Figure 2: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with vaccination greater than $v = \psi \left(1 - \frac{1}{R_0} \right)$

Figure 3: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with vaccination $v = [0, \psi - \frac{\psi}{R_0})$

Figure 4: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with persistent control

Figure 5: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with control aimed at asymptotic eradication

Summary of Vaccination and Disease Eradication: A Dynamic Analysis

This paper analyzes the conditions for eradication of diseases in a static and dynamic framework. In static framework, it investigates whether it is possible to eradicate diseases. There has been controversy in the public health research as to the possibility of eradication of diseases. Although epidemiologists have always felt that eradication of disease possible, there has been suspect among economists. A representative view of the suspect has been reported in the Hand Book of Health economics as “...although the introduction of a vaccine usually produces a sharp drop in the occurrence of a disease, the eradication of vaccine preventable diseases predicted by many at the time of these inventions has not been achieved except for smallpox. Of the roughly forty vaccines on the market, only the smallpox vaccine has eradicated its target disease. Diseases ... persist, despite explicit governmental efforts to eradicate them, and recent attempts to develop a vaccine against HIV or AIDS raise important questions about the causes behind these difficulties...” Revisiting the economics behind possibility of disease eradication in static framework, it seems that given the necessary economic incentives are fulfilled, in contrast to some of the recent literature; the results imply that it is possible to eradicate diseases, at least those whose dynamics follows Susceptible-Infective-Recovery, SIR framework. Individuals choose the level of vaccination that ensures eradication if the full cost of the vaccine that accrue to the individual is subsidized.

By expanding the framework to dynamic analysis, in the SIR models of infectious diseases, it tries to see whether there is biological limitation to eradication of infectious diseases. The canonical model of human disease dynamics imply that given certain critical level of vaccination is satisfied, it is possible to eradicate diseases. Given biological conditions are met and economic incentives are satisfied, unlike the suspect of some economists, it is possible to eradicate diseases.

Knowing that it is possible to eradicate infectious diseases, the social planner may resort to various public health policies including doing nothing, controlling, eliminating, eradicating or extinctions of diseases. But in order to make the choice of policies simpler, this paper considers two policies, namely doing nothing versus eradication, and considers them in their extreme version in light of economic benefits. It compares whether eradicating as fast as possible is preferable to doing nothing. The ultimate comparison involves choosing between the discounted cost of infection if the disease is left uninterrupted by deliberate policy measures and the discounted cost eradicating the disease as fast as possible. In relation to economic optimality, it has been shown that, in the limiting case where the ratio of terminal to initial infection approaches zero, eradicating as fast as possible is preferred to doing nothing when the discounted cost of infection of the fraction of initial population infected is greater than the unit cost of vaccination.

Depending on the parametric values, some diseases could be candidate for eradication and others may not be. Especially it seems that the cost benefit rule looks more sensitive to the ratio of the maximum fraction of infected people in which the disease can't expand to the equilibrium fraction of infected people if the disease is left uninterrupted. The cost benefit rule suggests fewer diseases to be eradicated as the ratio of terminal rate of infection to initial rate of infection approaches zero. In this regard the paper is in center view of epidemiologists that suggest eradication of most diseases and suspicious economists that suggest diverting our attention fully from eradication.

Finally, there has been some effort committed to see whether the current global action to eradicate polio is economically preferable. The results that follow from the cost benefit rule suggest that eradicating polio even using oral live-attenuated polio vaccine [OPV] is not optimal in both rich and poor countries.

I. Introduction and Background of the Research

Human diseases have long been on earth nearly as much as humanity. Like any other bad social outcomes, they have been more concentrated in poor nations especially in the tropics. The 1997 WHO world report pointed out that in 1996 alone, there were around 14 million deaths caused by infectious diseases only. It has also been noted by the report that infectious diseases are the primary causes of worldwide death. About two deaths out of four in poor countries are caused by infectious diseases. Lederberg (2003) pointed out that measles alone causes 900,000 deaths worldwide each year on average, which is 30% of the total death from vaccine preventable diseases. Due to AIDS, about 3 million individuals had lost their lives by 2004, and another 40 million people were living with the disease throughout the world¹. AIDS has left shambling economies and a large number of orphans to the extent that the social mechanism is unable to bear in the near future.

According to Diamond (1998), diseases have shaped the historical paths in which different nations evolved into where they are now. Historically there have been a number of attempts to have the world free of diseases though success has been much farther away from our site. Alternatively, extensive efforts have been devoted to control, elimination, and eradication of feasible diseases at least since the Dahlem's International Workshop on infectious diseases².

Previous researches in health economics established the view that eradication of disease is neither feasible nor optimal. However, such a view has been revised by recent researches. Recent researches have established that there are cases for eradication, if feasible, to be optimal in static framework. As will be discussed in next sections, eradication involves higher short run cost and everlasting long run pie, while control of diseases involves persistent costs. Eradication, if

¹ <http://www.avert.org/worldstats.htm>

² See Dowdle (1998) for instance.

biologically and operationally feasible, involves higher “dividend” compared with persistence high control. Hence, the prime motivation for the research is to investigate whether there is a case for feasibility of eradication, followed by the questions “under what conditions eradication is cost effective health policy?”

Public health policies have all been in the center of academic discussion. Depending on the nature of diseases, nations may resort to do nothing, control, eliminate, or eradicate diseases using treatment, sanitations, and vaccination. In simple SIR model of disease dynamics, individual decision to buy the level of protective measures that ensures control or eradication depends on its full cost. Under positive cost of vaccination, individuals have incentive to demand the level of vaccination so that the proportion of vaccinated people is below the critical level that eradicates the disease. On the other hand, if the whole cost of vaccination is subsidized, the proportion of people vaccinated matches with the necessary level of vaccination that eradicates the disease.

In dynamic framework, quick eradication is preferred to doing nothing when welfare cost of infection is the critical value of the welfare cost of infection. The critical value of the welfare cost of infection is a decreasing function of the ratio of the initial fraction of infection to terminal fraction among other parameters. If eradication is preferred, there is unique time of eradication. It is given by the logarithm difference between initial fractions of infected people less the tipping point discounted by the aggregate rate of withdrawal from fraction of infected individuals. Testing for polio, the cost benefit rule derived here implies that eradication of polio is not economically optimal using OPV or inactivated polio vaccine, IPV. The cost benefit rule suggests smaller number of diseases to be eradicated than comparable researches in the literature.

The paper is organized as follows: The introduction section is followed by the section in which the possibility of disease eradication is revisited in a static framework. Then dynamic models of disease evolution are overviewed along

different public health policies. Section four develops an alternative approach to the model forwarded by Barrett and Hoel (2004) [BH], and extracts results for economic optimality and applies the results of the model to the eradication attempt of polio. The final section pinpoints the limitations and possible ways of extending the analysis.

II. Is Eradication of Disease Possible?

The previously held belief of epidemiologist about infectious diseases, particularly the possibility of disease eradication, has been shaken in late 1990s with series of papers in economic epidemiology. Philipson (2000) summarizes the literature on economic epidemiology. Economic epidemiology is a phrase associated with systematic study of the interaction between biological epidemics and human behavior to achieve social objective. The key variable in economic epidemiology is prevalence elasticity which is the response of people's behavior due to changes in prevalence of diseases. Consideration of behavioral responsiveness would result in a number of distinct predictions concerning the incidence of diseases and its social cost than what the biological epidemiology suggests.

A number of researches in economic epidemiology have investigated the appropriate policy responses to infectious diseases. The most relevant question to this paper that they addressed is "Is eradication of diseases possible and/or is optimal?" Geoffard and Philipson (1997) formally argued that eradication is neither optimal nor possible in feasible markets under any form of expectations. Forceful argument, claiming eradication is impossible even under price subsidies, was forwarded by Geoffard and Philipson (1997).

"...price subsidies alone will not bring about eradication for the same reasons that price reductions through increased competition will not..."³

This argument goes counter to the crux and title of this paper. One is inclined to ask "if eradication is not possible, why do we bother about its optimality?" I will revisit, following Barrett (2003), the question on possibility of disease eradication

³ Barrett (2003) showed that under competitive market structure with positive prices global eradication of diseases is not possible.

at least by using subsidies in static setting below. Latter, I will extend it to dynamic settings.

I ignore the interaction between private agents in the market for vaccines and consider the decision as if taken by benevolent social planner that maximizes the benefit of the society taking into account all generations. The immediate implication of this assumption is that there is no difference between elimination and eradication in the analysis. Hence, the analysis in the rest of the paper is applicable to the world as a whole or to the last country making eradication decision after all other countries have eliminated the disease.

Although impossibility of eradication of diseases under positive price with private actions is robust, it is possible to eradicate a technically feasible disease with subsidies that cover the full cost of vaccine that accrue to individuals such as monetary cost of vaccination, the cost of time, pain, and the risk of getting infected from bad batch of vaccine. Given that the full cost of vaccine is covered, there is a possibility, as formally shown below for eradication to be feasible and optimal for an individual.

Epidemiology

For simplicity, I will assume the world as if ruled by a benevolent social planner. This assumption is a bit strong but it is a useful simplification to address the problem at hand. The relationship between the number of secondary infections, R , and proportion of people vaccinated, N^v and the rate of secondary infections, R_0 ⁴ is assuming homogenous mixing of population is given by (1).

⁴ It measures the average secondary infections when an infected person is introduced in to susceptible hosts. For a disease to spread, it is essential that $R_0 > 1$ as has been proved by many authors for instance Philipson (2000) or Hethcote (2000). Barrett (2003) reported $R_0=6$ for polio. The value of R_0 reported by Keeling (2001) range from 2 – 5 for AIDS, 3 – 5 for Smallpox, 16 – 18 for Measles, and more than 100 for Malaria.

$$R = (1 - N^v)R_0 \dots\dots\dots (1)^5$$

The proportion $(1 - N^v)$ represents the fraction of the population that is susceptible. Equation (1) says the number of secondary infections is equal to rate of secondary infections multiplied by the part of the population that is not immunized. We assume the vaccine ensures complete immunity and the disease is non-fatal. Then, it follows that for all diseases with $R_0 > 1$ the critical proportion of vaccination which ensures complete immunity is obtained when $R=1$ or smaller.

$$N^{vc} = 1 - \frac{1}{R_0} \dots\dots\dots (2)$$

The implicit assumption here is that the disease dies out itself once we vaccinate N^{vc} proportion of people. This may be due to herd immunity. By herd immunity we mean the immunity that individuals access without actually buying immunity services when other individuals have bought the immunity through vaccines or early accusation of disease. When some people demand immunity, the proportion of people infected decreases thereby decreasing the probability of getting infection for individuals. Gain in immunity that follows from such a positive externality due to decreased force of infections can be disease induced or vaccine induced. The herd immunity considered in this paper is induced by vaccine. Hethcote (2000) and Anderson and May (1991) reported estimates of N^{vc} for various diseases. For smallpox and polio, it is 0.80 while it is 0.86 for rubella, 0.89 for mups, .99 for malaria, *P. falciparum* in hyperendemic regions, and 0.94 for measles⁶. However, an infinitesimal decrease of fraction of people vaccinated from N^{vc} would result in the reemergence of the disease since the diseases we have considered have $R_0 > 1$.

⁵ See Anderson and May (2001) & Barrett (2003)

⁶ Alternatively, one can calculate N^{vc} from equation 2 using the rate of secondary infections.

Individual Decision

Individuals make decisions to demand immunity by comparing the expected cost of immunity to that of the disease. An individual can obtain immunity in two different ways: by directly purchasing the vaccine or from herd immunity. Let all the costs of the vaccine that accrue to the individual be “ c ”. The cost function of buying immunity is given by:

$$C_I = c \dots \dots \dots (3)$$

On the other hand, if an individual doesn't demand immunity then the expected cost of the disease is given by probability of acquiring the disease, which is the force of infection, times the total cost of infection. The force of infection is given by:

$$\lambda = R_0 \beta \left(N^{vc} - N^v \right)^7 \quad \forall N^v : 0 < N^c \leq N^{vc} \text{ and } \lambda = 0 \text{ otherwise } \dots \dots \dots (4)$$

The value β represents the transmission parameter, and the expected cost of disease is given by the product of the force of infection and the cost of infection, which is:

$$C_d = \lambda b = R_0 \beta b \left(N^{vc} - N^v \right) \dots \dots \dots (5)$$

The value b is the total cost of infection that accrues to an individual $\forall N^{vc} : N^{vc} \geq N^v$. Individuals have incentive from buying immunity so long as $C_I \leq C_d$, while they have incentive to do nothing when $C_I \geq C_d$. The state of interior competitive equilibrium, here, is defined as the state where the individual is indifferent between doing nothing and taking preventive measures. It is a state where individuals have no incentive to deviate. The competitive

7 Anderson and May (2001)

equilibrium, after little algebra, is given by (6) where the fraction N^{vm} is market induced proportion of population vaccinated.

$$N^{vm} = N^{vc} - c / b\beta R_0 \dots \dots \dots (6)$$

Under positive cost of vaccination the proportion of people vaccinated is below the tipping proportion which ensures full immunization. But, as long as the individual cost is zero, the proportion of individuals vaccinated equals the critical proportion which ensures complete immunity of the whole population for non-fatal diseases whose $R_0 > 1$ ⁸.

In fact, Lederberg (2003) begins the summary and assessment of Dahlem's international conference by pointing out that

"... The successful smallpox campaign demonstrates that global eradication of disease is possible, given the necessary technical base, political commitment, and economic resources for immunization and continued surveillance...."

With subsidized cost of vaccination, the next question is whether it is optimal to do so at least from the perspective of benevolent social planner. As described earlier, the social planner definitely incur higher cost in the short run by subsidizing full cost of vaccination so that the equality between critical proportion of vaccination and actual vaccination proportions is maintained. However, as the disease disappears just after such a cost is incurred, there is a long run pie from reduced burden of disease which includes disease induced costs and distortions. It is then optimal, to eradicate a specific disease if its discounted pie is greater than the cost of eradication. In deed, Barrett (2003) has formally addressed this question. In this paper, I will consider the problem in a dynamic setting.

⁸ Given $c \in [0, \infty)$, $c = 0 \Rightarrow N^{vc} = N^{vm} \wedge \forall c \in \mathbb{R}_+ \Rightarrow N^{vm} < N^{vc}$ ■

III. Dynamic Models of Infectious Diseases and Health Policies

Overview of Dynamic Models

Since the emergence of the first attempt by Daniel Bernoulli to formulate model for smallpox in 1760, modeling the dynamics of infectious diseases has been an important part of academic research. A number of models have been developed to describe and predict disease dynamics depending on the movement in to compartments of horizontal or vertical incidence.

When mothers are infected by a given communicable disease, some IgG antibodies are transferred to her child through placenta, which would transfer temporary passive immunity to infection to the child. Let M be the infants with passive immunity. When the temporary antibodies disappear, the infants join the class of susceptible designated by S. Moreover, newborns without antibodies also constitute this class. When there is a sufficient contact between an infected and susceptible, the disease transmits to the susceptible. With the transmission, a person joins exposed class E in the latent period. In this class, the person is infected but not infectious yet. When the latent period ends, the exposed enter the infective class I. Individuals in this class are not only infected but also transmit the disease to others given sufficient contacts are ensured. With the end of the infectious period, the individual enters recovered class R which includes those who gained permanent or temporary infection induced immunity.

Based on disease compartments there are about ten different models of disease dynamics. They are MSEIR, MSEIRS, SEIR, SEIRS, SIR, SIRS, SEI, SEIS, SI, and SIS⁹. For instance, the acronym SIR involves class of diseases in which Susceptible individuals are infected with adequate contact with the introduction of an infective person and gain permanent disease induced immunity in entering Recovered class, i.e. Susceptible \rightarrow Infected/infectious \rightarrow Recovery. On the other hand, the acronym SIS involves diseases in which individuals are

⁹ Again based on time, these models can be epidemic or endemic.

susceptible due to the gain in temporary infection-acquired immunity once they are infected i.e. Susceptible \rightarrow Infected/infectious \rightarrow Susceptible. The mathematical properties and essential results of most of these models are summarized by Hethcote (2000). Moreover, it was shown in the same paper that some of the most essential results of these models can be generated from the SIR model. In this paper, we exclusively focus on the simple SIR framework. Some of the examples of diseases whose dynamics was represented by SIR model are smallpox, influenza, chickenpox, polio, measles, and rubella. Let's see the dynamics in SIR model in a little detail below.

Dynamics in Endemic SIR Model

Many papers have investigated the dynamics in canonical endemic SIR models. However, the purpose here is to review the dynamics and see the biological conditions for feasibility of disease eradication. In the endemic SIR model we first specify how fractions of susceptible, infected, and immune individuals evolve over time.

The canonical SIR model, which is reported by Anderson and May (1991), provides simple picture of the dynamics of immunization in the epidemiology literature. The Model which we follow here is given by (7) and (8):

$$\dot{S}(t) = [\psi - \nu(t)] - [\psi + \lambda(t)]S(t) \dots \dots \dots (7)^{10}$$

Where $S(t)$ is the proportion of the people that is susceptible, $\lambda(t)$ is the rate at which susceptible individuals become infected, while $\nu(t)$ represents overall rate

¹⁰ We follow Barrett and Hoel (2003) in how we specify the dynamics when vaccination is introduced. We simply add the negative of the rate of vaccination into the differential equation of susceptible. This formulation is slightly different from some papers in the literature say Philipson (2000) or Andersen and May (1991). The difference is that in this model everyone of any age can be susceptible while in others it is only new births that do not demand immunity through vaccination that join the stock of susceptible. For our task at hand, however, both formulations give qualitatively the same result.

of vaccination. We assume that the disease is non-lethal and population is constant in which ψ refers to both death and birth rate.

Equation (7) implies that change in the stock of susceptible is given by birth rate ψ , increase in stock of susceptible, less withdrawal from the stock due to mortality, $\psi S(t)$, less withdrawal to infective, $\lambda(t)S(t)$, minus the fraction of individuals that demand immunity through vaccination.

The rate at which infected people immune from the non-lethal disease is given by δ and R_0 is the basic reproductive rate of the disease pathogen. The fraction of population infected under the control program $y(t)$ progress according to:

$$\dot{y}(t) = \lambda(t)S(t) - (\delta + \psi)y(t) \dots \dots \dots (8)$$

The first term in the right hand side represents the entry to the infected and the second term represents the withdrawal from being infected either due to natural recovery or mortality. This implies that the net change in the stock of infected is the difference between the entry and the withdrawal to the group. Assuming homogeneous mixing, that is the rate of infection $\lambda(t)$ is proportional to the infected population under the control program; i.e. $\lambda(t) = \beta y(t)$, where β is transmission parameter, and defining basic reproductive rate of micro parasite based on type II survival¹¹ by (9) the dynamics is provided by equations (7) and (8).

$$R_0 = \frac{\beta}{(\psi + \delta)} \dots \dots \dots (9)$$

$$\dot{S}(t) = f(S(t), y(t)) = \psi - v(t) - [\psi + \beta y(t)]S(t) \dots \dots \dots (10)$$

11 Andersen and May(1991)

$$\dot{y}(t) = g(S(t), y(t)) = (\delta + \psi)(R_0 S(t) - 1)y(t) \dots\dots (11)$$

Steady state requires both the time derivative of $S(t)$ and $y(t)$ are zero. Hence, from (10) and (11) with $\dot{S}(t) = 0$ and $\dot{y}(t) = 0$, at steady state, we have:

$$S^*(t) = \frac{\psi - \nu(t)}{\psi + \beta y(t)} \text{ \& } S^*(t) = \frac{1}{R_0} \dots\dots\dots (12)$$

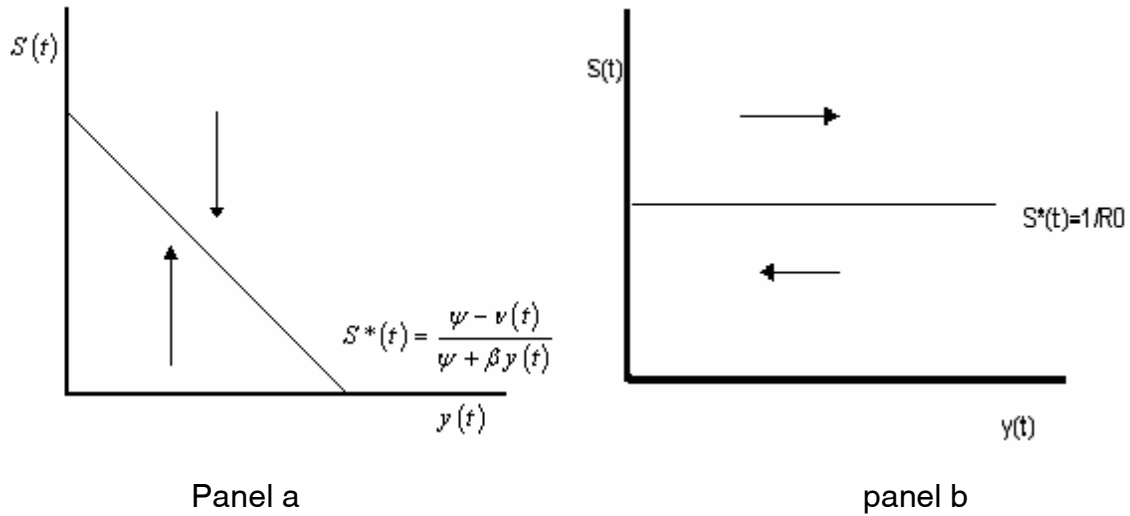


Figure 1: Phase Diagrams for Susceptible and Infected Individuals for a constant vaccination

As can be inferred from the figure in panel a, at steady state there is an inverse relationship between $S(t)$ and $y(t)$. Beginning from $f(S(t), y(t)) = 0$, if $S(t)$ increases, equation (10) implies that $S(t)$ decreases with time keeping ψ and $\nu(t)$ constant. An increase in $\nu(t)$ shifts $f(S(t), y(t)) = 0$ inward and decrease shifts it upward. Panel b shows the relationship between fraction of people infected and susceptible from $g(S(t), y(t)) = 0$. From the balanced path, if $S(t)$ increases, $g(S(t), y(t))$ implies that $y(t)$ increases and if $S(t)$ decreases, $y(t)$ decreases.

The $S(t)$ intercept of $f(S(t), y(t)) = 0$ is less than one, namely $S(t) = \frac{\psi - v(t)}{\psi} = 1 - \frac{v(t)}{\psi}$ with corresponding rate of vaccination given by (13). It is found by inserting $S^*(t) = \frac{1}{R_0}$ to $S^*(t) = \frac{\psi - v(t)}{\psi + \beta y(t)}$ and solving for v at the $S(t)$ -intercept of $f(S(t), y(t))$.

$$v^* = \psi \left(1 - \frac{1}{R_0} \right) \dots\dots (13)$$

We can have at least two cases that happen in practice. The value of the $S(t)$ intercept of $f(S(t), y(t)) = 0$ may be greater than or less than the value of the $S(t)$ intercept of $g(S(t), y(t)) = 0$. If vaccination is large enough to keep the $S(t)$ intercept of $f(S(t), y(t)) = 0$ below the $S(t)$ intercept of $g(S(t), y(t)) = 0$, then dynamics imply the possibility of eradication as can be seen from the figure below.

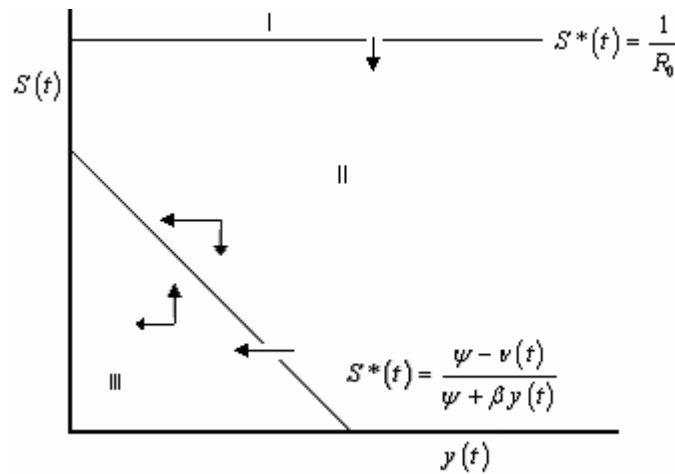


Figure 2: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with Vaccination

The directional arrows in the above phase diagram show how the fraction of infected population develops over time. They imply that any vaccination rate greater than or equal to $v = \psi \left(1 - \frac{1}{R_0}\right)$ ensures asymptotic eradication of the disease whether in initial point is in region I, or II, or III. If the initial point is in region I, the path moves down and when it hits the $S^*(t) = \frac{1}{R_0}$ line decreases vertically to the second region. Once in the second region, for that matter for any initial point in the second region, the integral trajectories move to the direction of the origin ensuring the intersection of the trajectories with the line $S^*(t) = \frac{\psi - v(t)}{\psi + \beta y(t)}$. When any path hits the line it moves horizontally to the left and there after to the direction of North-left which imply that $y(t) \rightarrow 0$ as $t \rightarrow \infty$! Hence with vaccination, it is biologically feasible to asymptotically eradicate disease following SIR model. In the previous section, it has been shown that when individuals optimize on their decision concerning disease, the biological feasibility can be accompanied by economic feasibility.

If the fraction of vaccinated individuals is not big enough to keep the $S(t)$ intercept of $f(S(t), y(t)) = 0$ below the $S(t)$ intercept of $g(S(t), y(t)) = 0$, then the dynamics resembles the case without vaccination.

When vaccination is zero, the $S(t)$ intercept of $f(S(t), y(t)) = 0$ is 1 since $y(t) = 0$ while the $S(t)$ intercept of $g(S(t), y(t)) = 0$ is always below 1 since R_0 is greater than one by assumption. Thus, the two phase diagrams intersect in one interior point with stable equilibrium of positive fraction of infected individuals. Practically the disease will not eradicate itself if there are no human preventive actions in operation.

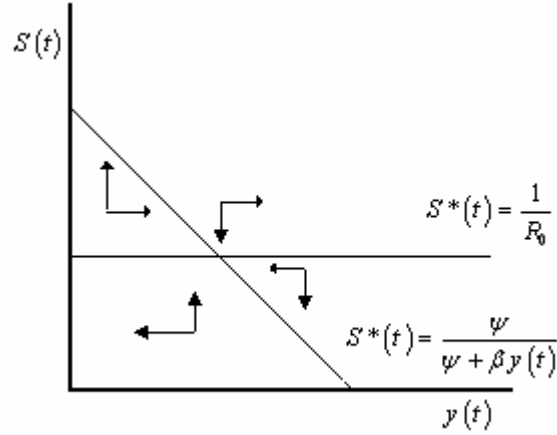


Figure 3: Phase Diagrams for Dynamics of Susceptible and Infected Individuals

The equilibrium point $(S^*(t), y^*(t))$ is locally asymptotically stable as it satisfies Lyapunov's condition for local stability. To see this,

for $J = \begin{pmatrix} f_1(S^*(t), y^*(t)) & f_2(S^*(t), y^*(t)) \\ g_1(S^*(t), y^*(t)) & g_2(S^*(t), y^*(t)) \end{pmatrix}$, it is the case that

$f_1(S^*(t), y^*(t)) + g_2(S^*(t), y^*(t)) < 0$ and $|J| > 0$ which is sufficient for local stability. However, the equilibrium is not globally asymptotically stable since it

doesn't satisfy Olech's theorem; i.e. $J' = \begin{pmatrix} f_1(S(t), y(t)) & f_2(S(t), y(t)) \\ g_1(S(t), y(t)) & g_2(S(t), y(t)) \end{pmatrix} \forall S(t) \in \mathbb{R}_+^2$ and

$y(t) \in \mathbb{R}_+^2$ the condition $|J'| > 0$ and $f_1(S(t), y(t)) + g_2(S(t), y(t)) < 0$ is not satisfied.

Public Health Policies

Health outcomes are governed by the tools of health policy which include preventive tools such as sanitation and vaccination and curative tools, mainly treatment. Vaccination has attracted the attention of economists long ago as it involves public policy and externalities. The individual decision to vaccinate itself in an attempt to prevent oneself from illness, also benefits other individuals since vaccinated individuals will not transmit the disease any more. Unvaccinated individuals, on the other hand, will increase the likelihood of infection of the rest of population. Such an externality has diverse implication for individual decision

making and economic policy. In one way, it implies that the individual decision over vaccination is suboptimal compared with what is socially desirable. This idea was formally forwarded by Brito et al (1991) in a static framework. They showed that compulsory vaccination is inferior to market based, and both compulsory and market based vaccinations are below what a benevolent social planner would decide due to the existence of externalities.

However, this view has been challenged by some authors. Francis (1997) has shown that there exists at least a condition under which the vaccination by the market outcome is socially optimal, and there exist no need for public interventions engendered by externalities in a dynamic setting. Rational epidemic also implies that individual's response to preventive measures differs depending up on the fraction of susceptible population vaccinated. As a result, some people might not even prefer to get vaccinated or take preventive measures in general if the expected value of being caught by the disease is by far lower than the disutility they suffer from vaccination or any preventive measures¹².

Treatment has also been an important curative tool in health policy. However, compared with vaccination, there barely is research in economic epidemiology that considers treatments in health policy. The two researches we have so far, Goldman and Lightwood (2002) and Gersovitz and Hammer (2004), have investigated the importance of treatment for control and eradication of diseases.

Policy Options

With the policy tools at hand and the relative effect of disease, policy makers may resort to do nothing, control, elimination, eradication or extinctions of diseases. Doing nothing might be preferable when the disease resulted in insignificant fraction of infected people. When doing nothing, the whole dynamics of the SIR model without intervention applies. As a result, the planner

12 See Geoffard and Philipson (1997), Philipson (2000), Kessing and Robert (2002), Gersovitz (2003), Gersovitz and Hammer (2003) and Gersovitz and Hammer (2004) for further and detailed considerations of these issues.

is left with $y^*(t) = \frac{\psi}{\beta}(R_0 - 1)$ and $S^*(t) = \frac{1}{R_0}$ of fraction of infected and susceptible individuals. We take the above steady state values to be initial values for any intervention.

Control

One of the policy options that require interventions is control of disease. Control of disease involves reducing the prevalence, incidence, of and morbidity due to a disease to an acceptable proportion as a result of deliberate action. For instance, diarrhea diseases are controlled in many countries. Another option than control is elimination of the disease and infections. A disease is eliminated when the incidence of the specific disease reduced to zero proportion as a result of deliberate action in a given geographical area. For instance, Neonatal tetanus is the case in point. However, elimination of diseases is quite different from elimination of infections. Elimination of infections involves reducing the incidence of infections by a specific agent to zero proportion in a specific geographical location engendered by intentional public action. In case of elimination of infections, continued cares need to be taken to avoid reestablishment of transmission. In Yemen, for example, poliomyelitis incidence was almost eliminated but has recently been reemerged back from Nigeria. In all the above three cases, continued intervention measures are required so as to prevent the reemergence of the disease outbreak.

There are number candidate paths of vaccination for control. Some of the paths involve control over the short run and simultaneously aim at asymptotic eradication. On the other hand, the rest of the control measure aim at reducing the fraction of infected people at “reasonable” positive value. This involves persistent vaccination effort and doesn’t lead to long run eradication. The critical proportion of vaccination that ensures asymptotic eradication is implied in the dynamics of the SIR model. Any vaccination constant at a value greater than or

equal to $v = \psi \left(1 - \frac{1}{R_0}\right)$, see equation 13, results in asymptotic eradication while below it results in reduced fraction of infected people but requires persistent control. Of all constant vaccination, the smallest vaccination that ensures asymptotic eradication is $v = \psi \left(1 - \frac{1}{R_0}\right)$. However, Anderson and May (1991) argue that the likely value is to continue vaccinating the new born ψ , since it is assumed to be harder to vaccinate adults.

Depending on social preference, the planner may aim at persistent control without asymptotic eradication or control aimed at asymptotic eradication. In figure (4), the dotted vertical lines represent the fraction of infected people without vaccination and with vaccination. In this figure, post vaccination dynamics is the same as that of pre-vaccination dynamics. On the other hand, the dynamics in figure (5) shows that the trajectory pre vaccination is different from that of post vaccination. In figure (5), since $f(S(t), y(t)) = 0$ curve sufficiently shifts inward, asymptotic eradication is guaranteed.

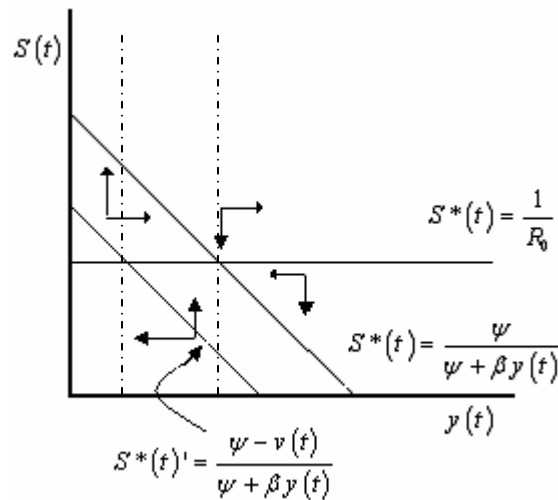


Figure 4: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with persistent control

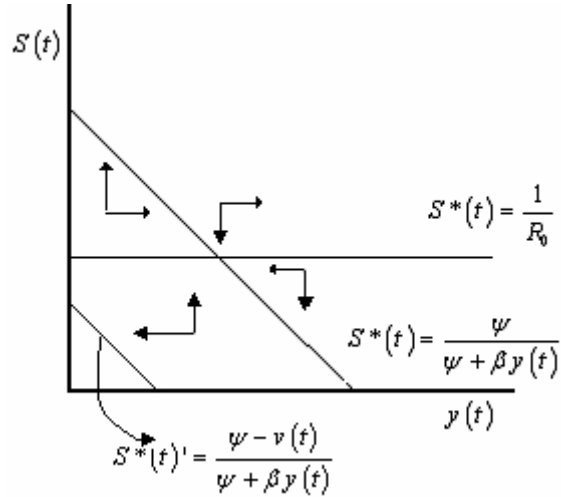


Figure 5: Phase Diagrams for Dynamics of Susceptible and Infected Individuals with control aimed at asymptotic eradication.

Eradication and Extinction

The two cases in which continuous preventions are no more required are eradication and extinction of diseases. As the name implies, eradication of a disease is achieved when the incidence of infection caused by specific agent is permanently reduced to zero proportion world wide. The global success story in eradication is smallpox. It took the world about ten years to eradicate smallpox. However, smallpox has not reached the state of extinction. A stage of extinction is reached when the specific agent of disease no longer exists in laboratory or nature. In fact, no disease has ever reached the state of extinction so far.

As eradication is a global public good, it requires strong coordination among nation states. Barrett (2003) has shown the conditions under which control and eradication of diseases are Nash equilibrium in a static framework. He argued that it may not be beneficial to eliminate disease unilaterally unless every other country eliminates. Or it may not be beneficiary for a last country to eradicate, though globally beneficiary, once every other nation has eradicated the disease. Depending on the conditions, global disease eradication may be a coordination game or prisoners' dilemma game. In the former case, eradication is more

plausible than the latter case of globally inefficient equilibrium. In the prisoner's dilemma equilibrium, each country resorts to control of disease through vaccination.

Since control and elimination of diseases and infections involve perpetual measures of control in general and in some cases vaccination, they involve unending costs. In addition to the cost of manufacturing and administering vaccination, there is a cost associated with the vaccine as there are some reactions to vaccines and distortions in consumption and production of exposures. However, once disease is eradicated, there is no more cost required due to vaccines and the disease; though it involves higher costs in the short run. As a result, there is unshrinking sustainable pie from eradication of disease compared with control due to cost savings. Moreover, there is also welfare gain from eradication caused by removal of disease induced distortions¹³. The pie may not be recognized once the disease disappeared as no one cares about smallpox now; but, its existence entails huge cost and diversion of resources as can be exemplified from the pandemic of AIDS.

Some examples are helpful at this point. For instance, assuming three percent of discount rate, BH reported that the benefit to cost ratio has been about 450:1 to the world while US alone benefited about \$5 billion from eradication of smallpox. For the case of poliomyelitis, BH reported that it exceeds \$900 per disability life year. Micro proportion evidence by Kim et al (1997) shown that the existence of Onchocerciasis in Teppi coffee plantation in Ethiopia decreased daily wage by 16% on average and labor supply by 4%.

Concerning the candidates for eradication, Barrett (2004) suggested that diseases that are already eliminated from rich nations are prime candidates. Globally, diseases such as malaria, poliomyelitis, measles, mums,

¹³ This point is clearly analyzed by Philipson (1995). According to this line of research, disease is considered as random tax on exposure and cost of illness covers only minor part of the welfare loss due to diseases.

dracunculiasis/Guinea worm, lymphatic filariasis, onchocerciasis/river blindness, chaga's disease, hansen's disease/leprosy, rubella, cysticercosis, and Sars are suggested to be eradicated as they have been eliminated from rich countries. Though there has been an effort to eradicate smallpox, measles, polio, and bovine tuberculosis globally, all of them except smallpox are not achieved so far. For instance, Sachs (2002) pointed out that global effort to eradicate malaria had begun 1955 and ended in 1960s and regarded as failure.

There have been a number of studies investigating why global disease eradication has failed for diseases tried so far such as measles, and bovine tuberculosis. Biological feasibility is precondition for elimination and eradication of diseases. Alyward et al (2000) and Nelson (1999) pointed out that eradication is a function of economic viability, biological capability, and social and political feasibility. The availability of effective vaccine and/or treatment, absence of non-human reservoirs, and the existence of practical diagnostic tools that single out the proportion and extent of infection that lead to transmission of the disease are the key factors determining biological feasibility. In this paper, we will focus on those diseases that are biologically feasible to be eradicated and concentrate on their economic optimality.

IV. Optimal Disease Eradication in Dynamic Framework

In this section, we compare the different health policy options we described earlier. As the crux under consideration is the dynamic benefit of disease eradication, the natural point to begin the analysis is on endemic epidemiological model.

The dynamics of disease predicted by rational epidemics is different from biological models. Biological models assume that behavior is prevalence inelastic. As a result, the probability of getting infected by an agent increases with prevalence of diseases. That is as more people are infected, there is more chance for the susceptible agents to get infection. However, if infection of an agent behavior is dependent, further consideration shows that such a result doesn't hold any more.

Most of the infectious diseases transmit through human actions. As the prevalence of a given disease increases, a rational agent resorts to protective measures by changing her behavior. Protection may involve vaccination for vaccine preventable diseases such as polio or measles. It may also be resorting to safe sex when the prevalence of sexually transmitted diseases increases, or getting the environment clean on the outbreak of cholera, etc. Such a protective behavior decreases the rate at which infection increases. Again behavioral response implies that agents may shun away from taking protective measures as new infection cases stop giving more chance for prevalence to increase at higher rate. Hence, a rational epidemic implies that the likelihood of transmission of diseases is no more constant under prevalence elastic behavior.

There are a number of options to eradicate diseases given the biological feasibility is satisfied. One way, which was described before, is to aim at asymptotic eradication. As the time of eradication approaches very large,

practically infinity, the pie from eradication diminishes. The pie from eradication is sizable if the disease is eradicated as quickly as possible.

In fact, there have been some attempts made in the literature to address this point before. For instance Barrett and Hoel (2003) have tried to explicitly address the problem. However, this paper is different from Barrett and Hoel's in the following respect.

Barrett and Hoel's simplify the biological dynamics of disease transmission as by a single differential equation in which instantaneous rate of force of infection per unit of time is proportional to the deviations from the steady state force of infection. Since this simplification involves epidemiological disease transmission mechanism, the simplification does not have any biological motivations. Since such an approximation to steady state needs biological justification, we propose a different way to tackle the problem that does not involve such difficulty of biological interpretation.

Here we follow an alternative approach to address this problem. We take (10) and (11) without any simplification and assume they hold for $y(t) > \varepsilon$, where epsilon is assumed to be very small number serving as critical or tipping point. Once the tipping point is achieved; i.e. $y(t) = \varepsilon$, the disease disappears quickly in its own mechanism without any more vaccination or social intervention. Hence, our results are not restricted to temporary deviations from steady state.

Equation (11) implies that eradicating as quick as possible involves two stages of processes. The first stage involves reducing the fraction of population that would have been susceptible with doing nothing to zero. The second stage involves reducing the fraction of infected population from $y(t) = y_0$ to $y(T) = \varepsilon$. But this can not practically be instantaneously. It is a gradual process that follows from

the dynamics of the fraction of individuals infected, once the fraction of susceptible population is reduced zero.

The common approach to measure the welfare cost of diseases is to take the cost of illness. According to this measure, higher morbidity inducing disease has a higher welfare loss¹⁴. Then the welfare optimization problem is to maximize the objective function given the discount rate of ρ is

$$W = \int_0^T e^{-\rho t} [-\omega v(t) - by(t)] dt \dots (14)$$

$by(t)$ represents the cost at time t of having a fraction $y(t)$ of the population infected, and $\omega v(t)$ ¹⁵ is the cost of vaccinating a proportion $v(t)$ of persons per unit of time (e.g., per year). T is the length of the vaccination program, which may be finite or infinite.

Note that the cost $\omega v(t)$ includes more than just the costs of the vaccination process and production of vaccines. In addition it includes costs related to side effects of vaccination. Moreover we also have additional constraints given below.

$$v(t) \geq 0 \dots (15)$$

14 Philipson (1995) presented, on the other hand, an alternative measure considering prevalence elastic behavior. According to this measure, disease is regarded as a stochastic tax on consumption of exposure. The prime case in point is AIDS. The price of sex consumption increases as the random tax on exposure, the incidence of the disease, increases. As the welfare loss of taxes is greater than their revenue effect, the overall welfare loss of diseases is greater than cost of illness. For a preliminary analyses and point of departure, we begin our analysis of welfare cost with cost of illness only.

15 More general cost function would be strictly increasing and convex. Without any simplification, it is not obvious how to proceed. One inevitably faces system of four non-linear differential equations with five variables which would make the task of extracting conditions for optimality of eradication difficult.

Further more, we assume the initial proportion of infected individuals is given by steady state proportion with zero vaccination. This assumption looks reasonable as the steady state is locally stable once the human intervention through vaccination is kept to zero. That is, we have:

$$S(t) \geq 0, S(0) = \frac{1}{R_0} \wedge y(0) = y_0 = \frac{\psi}{\beta}(R_0 - 1) > \varepsilon^{16} \dots (16)$$

$$y(T) = \varepsilon^{17} \dots (17)$$

Then the economic optimization, the social planner's problem is to maximize (14) subject to (10), (11), (15), (16), and (17).

Here instead of directly going for full fledged control problem, we compare doing nothing with eradicate as fast as possible¹⁸. Let w^0 be the welfare from doing nothing. It is achieved by keeping vaccination to zero and having consequent fraction of people infected. If the planner decides to do nothing infinitely in the future w^0 , is given by:

$$w^0 = \int_0^{\infty} e^{-\rho t} (-by_0) dt = -\frac{by_0}{\rho} = -\frac{b\psi(R_0 - 1)}{\beta\rho} \dots (18)$$

Note that the above equation is identical to BH's (C-1).

¹⁶ These initial values are the values that would have been steady state value had there not been intervention by vaccinating people.

¹⁷ Indeed keeping the terminal value of the fraction of individuals infected to zero doesn't qualitatively alter the results in the paper but it doesn't logically follow from our model.

With $S(t) = 0$, equation (11) reduces to $y(t) = \frac{\psi(R_0 - 1)}{\beta} e^{-(\delta+\psi)t}$ unless $\psi = 0$

¹⁸ Actually directly solving the control problem leads to "bang-bang" solution type which is qualitatively identical to the result presented here.

Fastest eradication, as implied in figure (2) or equation (11), is achieved in two stages assuming that there is no upper limit on $v(t)$. First the proportion of susceptible individuals has to be reduced zero. All susceptible should be vaccinated, and then the newborns should be vaccinated for some time until the proportion of infected individuals decreases to from initial state to epsilon. The cost of reducing the susceptible to zero is immediately is given by $\frac{\omega}{R_0}$, which is per unit of cost of vaccination ω multiplied by the initial fraction of susceptible $\frac{1}{R_0}$. With the fraction susceptible reduced to zero, the proportion of infected people evolves according to the differential equation (11) with $S(t) = 0$. This is given by:

$$y(t) = y_0 e^{-(\delta+\psi)t} \dots\dots\dots (19)$$

Thus, the welfare from eradication of as fast as possible, which is obtained by first keeping $S(t) = 0$ and then vaccinating the newborns till T , is achieved. It is given by, $-\frac{\omega}{R_0} - \int_0^T [\omega v(t) + by(t)] e^{-\rho t} dt$, inserting (19) and keeping $v(t) = \psi$ it reduces to:

$$w^1 = -\frac{\omega}{R_0} - \frac{\omega\psi}{\rho} (1 - \exp(-\rho T)) - \frac{b\psi(R_0 - 1)}{\beta(\psi + \delta + \rho)} (1 - \exp(-(\psi + \delta + \rho)T)) \dots (20)$$

Assuming epsilon equals to some fraction of the initial proportion of people infected, say $\epsilon = ky_0$, by substituting epsilon to (19), we have $ky_0 = y_0 e^{-(\delta+\psi)T}$ and

the time of eradication $T = -\frac{\ln k}{(\psi + \delta)}$ ¹⁹. Substituting the value of T in to (20), we have

$$w^1 = -\frac{\omega}{R_0} - \frac{\omega\psi}{\rho} \left(1 - k^{\frac{\rho}{\delta+\psi}}\right) - \frac{b\psi(R_0 - 1)}{\beta(\psi + \delta + \rho)} \left(1 - k^{\frac{\rho+\delta+\psi}{\delta+\psi}}\right) \dots (21)$$

When $k=1$, it is easy to see that the last two terms of equation (21) vanish and we remain with $w^1 = -\frac{\omega}{R_0}$ which is exactly equal to BH's equation (C-6) when relevant parameters are substituted.

Moreover, immediate eradication is preferred to doing nothing when $w^1 > w^0 \Leftrightarrow (21) > (18)$. Equivalently after substituting (9) and some algebra, we have:

$$\frac{\omega}{b} < \frac{\psi(R_0 - 1)}{(\psi + \delta + \rho)(\psi + \delta)} \left[\frac{(\psi + \delta + \rho) - \rho \left(1 - k^{\frac{(\psi+\delta+\rho)}{\rho}}\right)}{\rho + \psi R_0 \left(1 - k^{\frac{(\rho)}{\delta+\psi}}\right)} \right] \dots (22)$$

Equation (22) is our decision/cost benefit rule for eradication. Economic interpretation of (22) is hindered by nuisance combinations of parameters.

Alternatively, from (22), we can calculate the critical value of the welfare cost of the disease, b , which is the cost which will make the planner indifferent between doing nothing and aiming at “eradicating as quick as possible”, i.e.

¹⁹ $T \in \mathbb{R}_+ \Leftrightarrow k \in (0,1)$ Epsilon should be lesser than initial proportion of infected and this necessitates the value k to belong between zero and one exclusive.

$b^c = \underset{b \in \mathbb{R}_+}{Arg} \{w^1(\bullet) = w^0(\bullet)\}$ where \bullet represents respective parameters of each

equations. This critical value of b depends on k , so we write $b^c(k)$, which follows from (22):

$$b^c(k) = \frac{\omega(\psi + \delta + \rho)(\psi + \delta)}{\psi(R_0 - 1)} \left[\frac{\rho + \psi R_0 \left(1 - k^{\frac{(\rho)}{\delta + \psi}}\right)}{(\psi + \delta + \rho) - \rho \left(1 - k^{\frac{(\psi + \delta + \rho)}{\rho}}\right)} \right] \dots (23)$$

The cutoff/critical b in (23) determines our cost benefit rule. As mentioned above, (23) equals to b_{BH}^C for $k=1$.

$$b^c(1) = \frac{\omega(\psi + \delta)\rho}{\psi(R_0 - 1)} \dots (24)$$

Equation (24) is equivalent of Barrett and Hoel's equation (25) when relevant parameters are substituted.

When epsilon gets closer to zero, i.e. when $k \rightarrow 0$, our model gets closer to Anderson and May (1991)'s in which eradication becomes more and more difficult. As the limit of $k=0$ eradication is only possible asymptotically, and

$$b^c(0) = \frac{\omega(\psi + \delta + \rho)(\rho + \psi R_0)}{\psi(R_0 - 1)} \dots (25)$$

Since $b^c(k)$ is declining in k , the ratio between the highest and lowest possible ratio of the critical value of k is

$$\frac{b^c(0)}{b^c(1)} = \frac{(\psi + \delta + \rho)(\rho + \psi R_0)}{(\psi + \delta)\rho} \dots (26)$$

In the general case of $0 < k < 1$, eradicating as fast as possible is preferable to doing nothing provided the cost of infection is greater than the critical value given by $b^c(k)$. This critical value is higher than the critical value reported by Barrett and Hoel in their equation (25). The difference between the result reported here and that of Barrett and Hoel emanates from differences in w^1 . In their case, the cost of eradication is the cost of instantaneously reducing all the susceptible to zero. In our case, here however, there is more cost. We need to vaccinate the newborns till the proportion of infected individuals reduces to epsilon. Hence in our analysis, we have an additional sum of costs, namely the discounted cost of vaccinating the newborns plus the discounted cost of infection of the initially infected fraction (see also the discussion after (21)). Generally, equation (23) above provides higher cutoff b than Barrett and Hoel's.

In essence, our cost benefit rule is stronger. At least for the case of polio, approximating (10) and (11) by single equation that specifies the dynamics as deviation from locally asymptotically stable steady state significantly underestimates the value of “ b ” as $k \rightarrow 0$. Thus, Barrett and Hoel's simplification of the dynamics quantitatively affects the magnitude of results in the linear cost function. Let us briefly apply the result to the eradication of polio below and compare whether the difference is significant numerically for polio.

APPLICATION TO POLIO

Currently massive effort is being exerted to eradicate polio and dracunculiasis. In this section we will try to match the results of our model to eradication of polio.

The question is, then, whether (22) holds for polio for different values of k . To make comparisons, I will use the data used by Barrett and Hoel (2003).

There are two vaccines available in the market, OPV and IPV. Barrett and Hoel (2003) report that the marginal cost of OPV are \$46 in rich and \$4.88 in poor countries for complete immunity while that of IPV is around \$62 in rich and \$25 in poor countries. These give the estimates of ω in poor and rich countries.

Moreover, the value of ψ is estimated to be 3% in poor countries and 1% in rich countries. We assume ρ to be 3%. The other parameter we need to have is the recovery rate δ . The infectious period of polio is about 20 days and δ is approximately the inverse of this duration period times 364.25 to express δ per year ($\delta=18.21$) like the other parameters.

It is far difficult to have exact value of b since it amounts to the welfare cost of paralytic polio. Khan and Ehreth (2003) reported estimated cost of medical care only. It equals to \$25,000 and \$420 in rich and poor countries respectively. Instead in this section we use the cutoff value for value of b on (23), for different values of k , and compare it with the average value reported by Khan and Ehreth (2003). However, WHO (2002) reported the relative frequency of paralysis is 0.005, i.e., 1 out of 200 suffer paralysis. This implies the value of b to be 0.005 times the cost of single case of polio paralysis. With these, the cutoff values, which are determined by (23) and for different values of k , for both vaccine options are reported below.

Table 1: The cutoff Values of b for Polio using OPV and IPV for Rich and Poor Nations

k	OPV		IPV	
	Rich countries	Poor countries	Rich countries	Poor countries
1	100,574	3,560	135,557	18,240
BH's	100,795	3,568	135,854	18,280
0.1	101,502	3,647	136,808	18,684
0.01	102,262	3,728	137,831	19,097
0.001	103,019	3,808	138,851	19,508
0.0001	103,772	3,888	139,867	19,918
0.00001	104,523	3,986	140,879	20,326
0.000001	105,271	4,047	141,888	20,733
0.0000001	106,017	4,126	142,892	21,138
0	302,220	24,964	407,340	127,890

It is obvious that when $k \rightarrow 0$, the critical value of b increases. From the results in the table, eradication of polio is not an optimal deal for both rich and poor in the relevant domain of k . As it was shown in foot note 19, in our model k can't be zero or one. Hence the critical values at $k=1$ and $k=0$ give the lower and the upper boundaries for the critical welfare cost of polio. This result is qualitatively consistent with Barrett and Hoel (2004) and different from Bart et al (1996) and Kahn and Ehreth (2003).

VI. Concluding Remarks

In this paper an attempt has been made to single out the conditions for eradication of infectious diseases in simplest framework. For those diseases following SIR framework, it is possible to eradicate diseases asymptotically. Moreover, when eradication is feasible, it is optimal to eradicate diseases when the critical cost of infection is lower than the cost of infection. It has been also checked that eradication of polio is not economically optimal using IPV or OPV.

The results of the model can be extended in various ways. One obvious limitation of the analysis here is it assumes constant vaccination effort. More fruitful analysis is to consider variable vaccination effort in the process of control and eradication.

Moreover, the analysis here considers the world as if it is one country. If we relax this assumption and consider interactions among countries, we can gain more insight. In addition to these, we assumed that vaccines can be supplied as they are required. However, we are totally abstracting from monopoly rent of vaccines patents, and complete loss of income for monopolist from the vaccine once the disease eradicated. Surely such interaction shapes the control we have over vaccination and the path of eradication.

The other more difficult extension in this framework can be made with the cost function. Here we assumed positive and constant marginal costs. But, if marginal costs are allowed to vary with the level of vaccination effort, it is not possible to accommodate the biological dynamics of SIR model. The fruitful direction to solve this problem is to consider discrete time formulation of the problem and go for numerical solutions.

References

- Aylward RB, Hull HF, Cochi SL, Sutter RW, Olive JM, Melgaard B. (2000) *Disease eradication as a public health strategy: a case study of poliomyelitis eradication*, **Bulletin of World Health Organization** 78(3):285-97
- Anderson, R.M. and R.M. May (1991), *Infectious Diseases of Humans: Dynamics and Control*, Oxford: Oxford University Press.
- Barrett, S. (2003), "Global Disease Eradication," **Journal of the European Economic Association**, 1: 591-600.
- _____ (2004), "Eradication versus control: the economics of global infectious disease policies". **Bulletin of World Health Organization**, vol.82, no.9, p.683-688.
- _____ and Michael O. Hoel (2004), *Optimal Disease Eradication*, **Nota di Lavarò** 50: FEEM
- Bart K. J, J. Foulds, and P. Patriaca (1996) "Global Eradication of Poliomyelitis: Benefit-Cost Analysis" **Bulletin of World Health Organization**, vol.74, no.1, p.35-45
- Brito, D. L., E. Sheshinski, and M. D. Intriligator (1991) "Externalities and Compulsory Vaccinations" **Journal of Public Economics** 45: 69-90
- Diamond Jared (1998), *Guns, Germs and Steel*, London: Vintage.
- Dowdle Walter R. (1998). *The Principles of Disease Elimination and Eradication*, **Bulletin of World Health Organization**; 76:22-5
- Francis, P. J. (1997)"Dynamic Epidemiology and the Market for Vaccinations" **Journal of Public Economics** 63: 383-406.
- Geoffard, Pierre-Yves., and T. Philipson (1997) "Disease Eradication: Private vs. Public Vaccination," **American Economic Review**. 87: 222–230.
- Gersovitz, M. (2003), "Births, Recoveries, Vaccinations and Externalities," in R.J. Arnott *et al.* (eds.), **Essays in Honor of Joseph Stiglitz**, Cambridge, MA: MIT, pp. 469-83.

- Gersovitz, M. and J.S. Hammer (2003), "*Infectious Diseases, Public Policy, and the Marriage of Economics and Epidemiology*", **World Bank Research Observer**, 10, No 2, pp 129 - 157.
- Gersovitz, M. and J.S. Hammer (2004), "*The Economical Control of Infectious Diseases*," **Economic Journal**, 114, No 492, 1- 27
- Goldman Steven Marc and James Lightwood (2002), "*Cost Optimization in the SIS Model of Infectious Disease with Treatment*" **Topics in Economic Analysis & Policy**, Volume 2, No 1
- Hethcote Herbert W. (2000) "*The Mathematics of Infectious Diseases*", **Society for Industrial and Applied Mathematics [SIAM] Review**, 42, No.4, 599–653
- Kahn and Ehreth (2003).....
- Keeling Matt (2001) "*The mathematics of diseases: One Parameter (almost) does it all*", + **Plus Online Magazine ... Living Mathematics**, Cambridge University, Issue 14, <http://plus.maths.org/issue14/features/diseases/index.html>
- Kessing, S. and R. Nuscheler (2002) "*Monopoly Pricing in the Market for Vaccines*," **Freie Universit"at Berlin working paper**
- Kim Aehyung, Ajay Tandon, Asrat Hailu, and Others (1997) "*Health and Labor Productivity: The Economic Impact of Onchocercal Skin Disease*", **Policy Research Working Papers, World Bank**
- Lederbeg Joshua (2003) *Considerations for Viral Disease Eradication: Lessons Learned and Future Strategies: Workshop Summary*, <http://books.nap.edu/catalog/10424.html>
- Nelson Ann Marie, "*The Cost of Disease Eradication: Smallpox and Bovine Tuberculosis*", **Annals of the New York Academy of Sciences** 894:83-91
- Philipson, Tomas (2000) "*Economic Epidemiology and Infectious Diseases*" In Joseph Newhouse and Anthony Culyer (eds.), **the Handbook of Health Economics**, North Holland Press
- Sala-i-martin Xavier (2002) "*Unhealthy People are Poor People...And Vice Versa*", keynote address at the European Conference on Health Economics of the International Health Economics Association, Paris, July

Sachs Jeffrey D. (2002) "A New Global Effort to Control Malaria," **Science**, Vol. 298, 4

Seirstad Atle (1988)"Sufficient Conditions in Free Final Time Optimal Control Problems", **SIAM Journal of Control and Optimization**, 26.

World Health Organization (2002), *Poliomyelitis, Fact Sheet*, No. 114